

# Weak Gravity Conjecture and Holographic Dark Energy Model with Interaction and Spatial Curvature

Cheng-Yi Sun<sup>\*,a</sup>

<sup>a</sup>Institute of Modern Physics, Northwest University,  
Xian 710069, P.R. China.

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## Abstract

In the paper, we apply the weak gravity conjecture to the holographic quintessence model of dark energy. Three different holographic dark energy models are considered: without the interaction in the non-flat universe; with interaction in the flat universe; with interaction in the non-flat universe. We find that only in the models with the spatial curvature and interaction term proportional to the energy density of matter, it is possible for the weak gravity conjecture to be satisfied. And it seems that the weak gravity conjecture favors an open universe and the decaying of matter into dark energy.

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**Key words:** weak gravity conjecture, holographic, quintessence

## 1 Introduction

Increasing evidence suggests that the expansion of our universe is being accelerated [1, 2, 3]. Within the framework of the general relativity, the acceleration can be phenomenally attributed to the existence of a mysterious exotic component with negative pressure, namely the dark energy [4, 5], which dominates the present evolution of the universe. However, we know little about the nature of dark energy. The dark energy problem has become one of the focuses in the fields of both cosmology and theoretical physics today.

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\*cysun@mails.gucas.ac.cn; ddscy@163.com

The most nature, simple and important candidate for dark energy is the Einstein's cosmological constant, which can fit the observations well so far. But the cosmological constant is plagued with the well-known fine-tuning and cosmic coincidence problems [4, 5]. Another candidate for dark energy is scalar-field dark energy model. So far, a wide variety of scalar-field dark energy models have been proposed, such as quintessence [6], phantom [7],  $k$ -essence [8], tachyon [9], quintom [10], hessence [11], etc. Usually, the scalar-field models are regarded as a low-energy effective description of the underlying theory of dark energy. Other dynamical dark energy models include Chaplygin gas models [12], braneworld models [13], etc.

A lot of efforts have been made to solve the dark energy problem, but no effort seems to be successful so far. Actually, it is generally believed that the dark energy problem is in essence an issue of quantum gravity. However a complete theory of quantum gravity is still unknown. Then it becomes natural for physicists to explore the nature of dark energy just in light of some fundamental principles of quantum gravity. The holographic principle is commonly believed to be such a principle [14]. Based on the principle, the holographic dark energy models have been suggested [15]. The model has been studied widely, and supported by various observations (see citations of Ref.[15]). And even, it is found that the holographic dark energy model is favored by the anthropic principle [16].

On the other hand, it is generally believed that string theory is the most promising theory of quantum gravity. Recent progress [17] suggests that there exist a vast number of semi-classical consistent vacua in string theory, named *Landscape* [18]. However, not all semi-classical consistent vacua are actually consistent on the quantum level, and these actually inconsistent vacua are called *Swampland* [19]. Self-consistent landscape is surrounded by the swampland. The weak gravity conjecture (WGC) is suggested to be a new criterion to distinguish the landscape from the swampland [20, 21]. The conjecture can be most simply stated as gravity is the weakest force. For a four-dimensional U(1) gauge theory, WGC implies that there is an intrinsic UV cutoff [20]

$$\Lambda \leq gM_p,$$

where  $g$  is the gauge coupling constant and  $M_p$  is the Planck scale. Furthermore, in [22], it is argued that WGC also indicates an intrinsic UV cutoff for the scalar field theories with gravity, e.g.

$$\Lambda \leq \lambda^{1/2}M_p$$

for  $\lambda\phi^4$  theory. In the slow-roll inflation model with the potential  $V(\phi) \sim \lambda\phi^4$ , Hubble constant can be taken as the IR cutoff for the field theory. Then the requirement that the IR cutoff should be lower than the UV cutoff indicates [22]

$$\frac{\lambda^{1/2}\phi^2}{M_p} \sim H \leq \Lambda \leq \lambda^{1/2}M_p, \quad \text{or,} \quad \phi \leq M_p \quad (1)$$

This leads the author in [23] to conjecture that the variation of the inflaton during the period of inflation should be less than  $M_p$ ,

$$|\Delta\phi| \leq M_p. \quad (2)$$

And it is found that this can make stringent constraint on the spectral index of the inflation model [23].

Recently, the criterion (2) has been used in [24] to explore the quintessence model of dark energy, in [25] to study Chaplygin gas models [12], and in [26] to study the agegraphic dark energy model [27]. Then if the holographic dark energy scenario is assumed to be the underlying theory of dark energy, and the low-energy scalar field can be used to describe it effectively [28], can the weak gravity conjecture, i.e.,  $\Delta\phi < M_p$ , be satisfied? In the direction, some work has been done [29, 30]. It is found that the holographic quintessence model does not satisfied the conjecture [30]. However, in [29, 30], only the non-interacting holographic quintessence in the flat universe is discussed. In [31], it is found that when simultaneously considering the interaction and spatial curvature in the holographic dark energy model, the parameter space is amplified much more. Then it may be possible for the weak gravity conjecture to be satisfied in the interacting holographic quintessence model with spatial curvature. Here we will discuss the problem.

In the paper, we will first recall the interacting holographic dark energy model in the non-flat universe. Then we will discuss the possible theoretical constraints on the holographic quintessence model from the weak gravity conjecture and try to find the possibility for the conjecture to be satisfied within the parameter space displayed in [31]. Finally, conclusion will be given.

## 2 Holographic Dark Energy Model with Interaction and Space Curvature

The Friedmann-Robertson-Walker (FRW) universe is described by the line element

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (3)$$

where  $a(t)$  is the scale factor, and  $k$  is the curvature parameter with  $k = -1, 0, 1$  corresponding to a spatially open, flat and closed universe, respectively. The Friedmann equation is

$$3M_p^2 \left( H^2 + \frac{k}{a^2} \right) = \rho_m + \rho_D, \quad (4)$$

where  $M_p = (8\pi G)^{-1/2}$ ,  $\rho_m$  is the energy density of matter and  $\rho_D$  is the energy density of dark energy. By defining

$$\Omega_k = \frac{\rho_k}{\rho_c} = \frac{k}{H^2 a^2}, \quad \Omega_D = \frac{\rho_D}{\rho_c}, \quad \Omega_m = \frac{\rho_m}{\rho_c}, \quad (5)$$

where  $\rho_k = k/a^2$  and  $\rho_c = 3M_p^2 H^2$ , we can rewrite the Friedmann equation as

$$1 + \Omega_k = \Omega_D + \Omega_m. \quad (6)$$

In the holographic dark energy model, the energy density  $\rho_D$  is assumed to be [32]

$$\rho_D = 3d^2 M_p^2 R_h^{-2}, \quad (7)$$

where  $d$  is a constant parameter,  $R_h = ar(t)$  and  $r(t)$  satisfies [33]

$$\int_0^{r(t)} \frac{dr}{\sqrt{1 - kr^2}} = \int_t^{+\infty} \frac{dt}{a(t)}. \quad (8)$$

For a closed universe,  $k = 1$ , from the equation above we have

$$\arcsin(\sqrt{k}r) = \sqrt{k} \int_t^{+\infty} \frac{dt}{a(t)}. \quad (9)$$

Together with Eq.(7), we have

$$\arcsin\left(\sqrt{k \frac{3d^2 M_p^2}{a^2 \rho_D}}\right) = \sqrt{k} \int_t^{+\infty} \frac{dt}{a(t)}. \quad (10)$$

The derivative of the equation with respect to  $t$  gives

$$\dot{\rho}_D = -3H\rho_D \times \frac{2}{3} \left(1 - \sqrt{\frac{\Omega_D}{d^2} - \Omega_k}\right). \quad (11)$$

The equation is obtained by using  $k = 1$ . But it can be easily checked that Eq.(11) holds in the cases of  $k = 0$  and  $k = -1$ , too. We can define an effective equation of state parameter  $w_D^{\text{eff}}$  by

$$\dot{\rho}_D + 3H(1 + w_D^{\text{eff}})\rho_D = 0. \quad (12)$$

Then comparing Eqs.(11) and (12), we have

$$w_D^{\text{eff}} = -\frac{1}{3} \left(1 + 2\sqrt{\frac{\Omega_D}{d^2} - \Omega_k}\right). \quad (13)$$

Using Eqs.(5), we may rewrite Eq.(11) as [31]

$$\sqrt{\frac{\Omega_D H^2}{d^2} - \frac{k}{a^2}} = \frac{\dot{\Omega}_D}{2\Omega_D} + H + \frac{\dot{H}}{H}. \quad (14)$$

Now consider some interaction between holographic dark energy and matter [31]

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (15)$$

$$\dot{\rho}_D + 3H(1 + w_D)\rho_D = -Q, \quad (16)$$

where  $Q$  denotes the phenomenological interaction term. In Ref.[31] three types of interaction are considered,

$$Q_1 = -3bH\rho_D \quad (17)$$

$$Q_2 = -3bH(\rho_D + \rho_m) \quad (18)$$

$$Q_3 = -3bH\rho_m. \quad (19)$$

Following Ref.[31], for convenience, we uniformly express the interaction term as  $Q_i = -3bH\rho_c\Omega_i$ , where  $\Omega_i = \Omega_D, 1 + \Omega_k$  and  $\Omega_m$ , for  $i = 1, 2$  and  $3$ , respectively.

From Eqs.(4), (15) and (16), we can obtain

$$w_D = -\frac{1}{3H\rho_D} \left( 2\frac{\dot{H}}{H}\rho_c - 2H\rho_k \right) - \rho_c - \rho_k. \quad (20)$$

Substituting the equation into Eq.(16) and using Eq.(5), we can have [31]

$$2(\Omega_D - 1)\frac{\dot{H}}{H} + \dot{\Omega}_D + H(3\Omega_D - 3 - \Omega_k) = 3bH\Omega_i, \quad (21)$$

where  $i = 1, 2$  and  $3$ . From Eqs.(14) and (21), finally we get the following two equations governing the evolution of the interacting holographic dark energy in the non-flat universe

$$\frac{d\tilde{H}}{dz} = -\frac{\tilde{H}}{1+z}\Omega_D \left( \frac{3\Omega_D - \frac{\Omega_{k0}(1+z)^2}{\tilde{H}^2} - 3 - 3b\Omega_i}{2\Omega_D} - 1 + \sqrt{\frac{\Omega_D}{d^2} - \frac{\Omega_{k0}(1+z)^2}{\tilde{H}^2}} \right), \quad (22)$$

$$\frac{d\Omega_D}{dz} = -\frac{2\Omega_D(1 - \Omega_D)}{1+z} \left( \sqrt{\frac{\Omega_D}{d^2} - \frac{\Omega_{k0}(1+z)^2}{\tilde{H}^2}} - 1 - \frac{3\Omega_D - \frac{\Omega_{k0}(1+z)^2}{\tilde{H}^2} - 3 - 3b\Omega_i}{2(1 - \Omega_D)} \right), \quad (23)$$

where  $1 + z = \frac{1}{a}$ ,  $\tilde{H} \equiv \frac{H}{H_0}$  and hereafter the subscript 0 denotes the present value of the corresponding parameter. And we have used  $\Omega_k = \frac{\Omega_{k0}(1+z)^2}{\tilde{H}^2}$  and  $a_0 = 1$ .

### 3 Holographic Quintessence Model and Weak Gravity Conjecture

For a single-scalar-field quintessence model with potential  $V(\phi)$ , the energy density and pressure of the quintessence scalar field are

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad (24)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \quad (25)$$

So the equation of state is

$$w_\phi = \frac{\rho_\phi}{p_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 + V}{\frac{1}{2}\dot{\phi}^2 - V} \quad (26)$$

From Eqs.(24) and (26), we can obtain easily

$$\rho_\phi = \frac{\dot{\phi}^2}{1 + w_\phi} \quad (27)$$

Without loss of generality, we may assume  $dV/d\phi < 0$  and  $\dot{\phi} > 0$ . Thus from Eq.(27), we may have

$$\dot{\phi} = \sqrt{(1 + w_\phi)\rho_\phi} \quad (28)$$

Then the weak gravity conjecture tells us [22, 24, 29, 30]

$$\begin{aligned} 1 &\geq \frac{|\Delta\phi(z)|}{M_p} = \int \frac{\dot{\phi}}{M_p} dt \\ &= \int_0^z \sqrt{3[1 + w_\phi(z')]\Omega_\phi(z')} \frac{dz'}{1 + z'}, \end{aligned} \quad (29)$$

where  $\Omega_\phi = \rho_\phi/\rho_c$ .

In the holographic quintessence model, the holographic dark energy is assumed to be described by the effective scalar field. Then naturally we have

$$\rho_\phi = \rho_D \Rightarrow \Omega_\phi = \Omega_D \quad (30)$$

Here following Ref.[34], we identify  $w_\phi$  with  $w_D^{\text{eff}}$ , instead of  $w_D$ ,

$$w_\phi = w_D^{\text{eff}} = -\frac{1}{3} \left( 1 + 2\sqrt{\frac{\Omega_D}{d^2} - \Omega_k} \right). \quad (31)$$

We must require  $d \geq 1$  so that  $w_\phi \geq -1$ . Substituting Eq.(31) into Eq.(29), then the weak gravity conjecture for the holographic quintessence with interaction and spatial curvature reads

$$1 \geq \frac{|\Delta\phi(z)|}{M_p} = \int_0^z \sqrt{2\left(1 - \sqrt{\frac{\Omega_D(z')}{d^2} - \frac{\Omega_{k0}(1+z')^2}{\tilde{H}^2}}\right) \Omega_D(z') \frac{dz'}{1+z'}}. \quad (32)$$

where  $\Omega_D$  and  $\tilde{H}$  can be obtained by numerically solving Eqs.(22) and (23) if the initial values  $\Omega_{m0}$  and  $\Omega_{k0}$ , and the values of the constant parameters  $b$  and  $d$  are given.

The case of  $\Omega_{k0} = 0$  and  $b = 0$  has been discussed in Ref.[29, 30], and it is found the weak gravity conjecture cannot be satisfied for a holographic quintessence model. Here we expect the weak gravity conjecture may be satisfied when both the interaction and spatial curvature are considered.

Since

$$\Omega_D = \frac{d^2}{H^2 R_h^2}, \quad (33)$$

then larger  $b$  indicates bigger  $\Omega_D$  and more difficult for Eq.(32) to be satisfied. On the other hand, we must require  $d \geq 1$  so that  $w_\phi > -1$ . So in the paper, we will focus on the case of  $d = 1$ . Then Eq.(32) becomes

$$1 \geq \frac{|\Delta\phi(z)|}{M_p} = \int_0^z \sqrt{2\left(1 - \sqrt{\Omega_D(z') - \frac{\Omega_{k0}(1+z')^2}{\tilde{H}^2}}\right) \Omega_D(z') \frac{dz'}{1+z'}}. \quad (34)$$

### 3.1 the non-interacting holographic quintessence model with spatial curvature

With  $b = 0$ , we can rewrite Eq.(34) as

$$1 \geq \frac{|\Delta\phi(z)|}{M_p} = \int_0^z \sqrt{2\left(1 - \frac{\Omega_{m0}(1+z)^\alpha}{\tilde{H}^2}\right) \Omega_D(z') \frac{dz'}{1+z'}}, \quad (35)$$

where  $\alpha = 3$ , and we have used  $\Omega_m = \frac{\Omega_{m0}(1+z)^\alpha}{\tilde{H}^2}$  and Eq.(6). Then the equation above tells us that larger  $\Omega_{m0}$  will make it easier for Eq.(35) to be satisfied. However, in Ref.[30], it is shown that Eq.(35) can not be satisfied in the flat universe even for  $\Omega_{m0} = 0.35$ .

Now let us consider the effect of the spatial curvature. From Eq.(6), we know that larger  $\Omega_k$  indicates larger  $\Omega_D$  and then makes it more difficult for Eq.(35) to be satisfied. So negative  $\Omega_k$  will make it easier for Eq.(35) to be satisfied than that in the flat universe. We plot the result

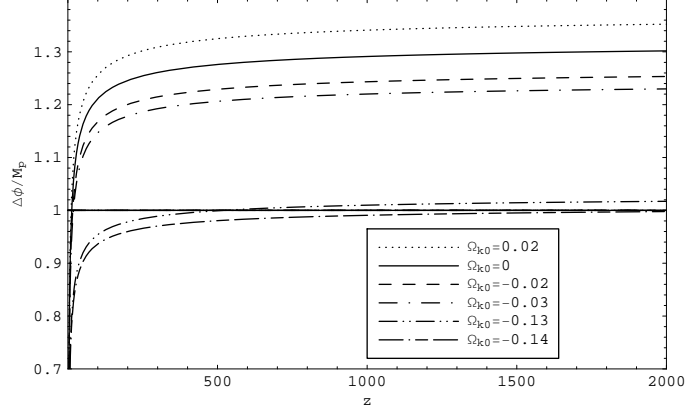


Fig. 1:  $\Delta\phi(z)/M_p$  versus the redshift  $z$  in the non-interacting holographic quintessence model with spatial curvature for fixed  $\Omega_{m0} = 0.34$  and different  $\Omega_{k0}$ .

of  $\Delta\phi/M_p$  versus the redshift  $z$  in Fig.1. In the case, we find that it is impossible for the weak gravity conjecture to be satisfied within the parameter space displayed in FIG.4 in [31]. Actually, in order to match the weak gravity conjecture, we should take  $\Omega_{k0} \lesssim -0.14$  which has been far outside the range of  $\Omega_{k0}$  displayed in FIG. 4 in [31]. In Fig.1, we have fixed  $\Omega_{m0} = 0.34$  which is slightly bigger than the maximum value of  $\Omega_{m0}$  displayed in FIG.4 in Ref.[31], since smaller  $\Omega_{m0}$  would make it more difficult for Eq.(35) to be satisfied.

### 3.2 the interacting holographic quintessence model without spatial curvature

With the interaction between the dark energy and matter, we can rewrite Eq.(34) as

$$1 \geq \frac{|\Delta\phi(z)|}{M_p} = \int_0^z \sqrt{2\left(1 - \frac{\Omega_{m0}(1+z)^{\alpha(z)}}{\tilde{H}^2}\right)} \Omega_D(z') \frac{dz'}{1+z'}, \quad (36)$$

where  $\alpha(z)$  is defined by

$$\ln \frac{\rho_m}{\rho_{m0}} = \alpha(z) \ln(1+z). \quad (37)$$

Here the Friedmann equation reads

$$1 = \Omega_m + \Omega_D.$$

Since the form of Eq.(36) is similar to that of Eq.(35), we can get a similar conclusion that larger  $\Omega_{m0}$  will make it easier for Eq.(36) to be satisfied.



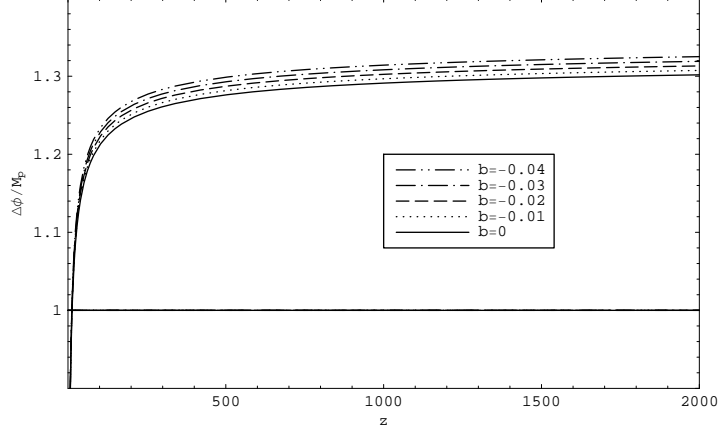


Fig. 2:  $\Delta\phi(z)/M_p$  versus the redshift  $z$  in the holographic quintessence model with the interaction term  $Q = Q_1$  in the flat universe for fixed  $\Omega_{m0} = 0.34$  and different  $b$ .

For the three types of interaction, we can uniformly express the conservation law Eq.(15) as

$$\dot{\rho}_m = -3H\left(1 + b\frac{\Omega_i}{\Omega_m}\right)\rho_m, \quad (38)$$

where  $i = 1, 2$  and 3. From Eqs.(37) and (38), we have

$$\alpha(z) = \frac{3}{\ln(1+z)} \int_0^z \left(1 + b\frac{\Omega_i(z')}{\Omega_m(z')}\right) \frac{dz'}{1+z'}. \quad (39)$$

Obviously,  $\alpha = 3$  for  $b = 0$ ,  $\alpha < 3$  for  $b < 0$  and  $\alpha > 3$  for  $b > 0$ . Or, in the other words, smaller  $b$  indicates smaller  $\alpha$ , and then will make it more difficult for Eq.(36) to be satisfied.

On the other hand, it has been shown in [30] that the weak gravity conjecture cannot be satisfied in the holographic quintessence models in the flat universe with  $b = 0$  even for  $\Omega_{m0} = 0.35$ . Since smaller  $b$  or  $\Omega_{m0}$  would make it more difficult for the weak gravity conjecture to be satisfied, and  $\Omega_{m0} = 0.35$  has been far beyond the range of  $\Omega_{m0}$  displayed in FIG.1 in Ref.[31], we can conclude that the weak gravity conjecture cannot be satisfied in the flat universe with non-positive  $b$ . Our conclusion is illustrated by the results displayed in Fig.2 and Fig.3.

For the models with the interaction term  $Q = Q_1$  or  $Q = Q_2$ , only the case of  $b \leq 0$  is regarded as the realistic physical situation. The reason is that in the two types of models, positive  $b$  will lead  $\rho_m$  to become negative in the far future. Then we know that, in the holographic quintessence models with interaction term  $Q = Q_1$  or  $Q = Q_2$  in the flat universe, the weak gravity conjecture can not be satisfied, as shown in Fig.2 and Fig.3.

If  $Q = Q_3$ , the conservation law of matter is

$$\dot{\rho}_m = -3H(1+b)\rho_m \Rightarrow \rho_m \propto a^{-3(1+b)}. \quad (40)$$

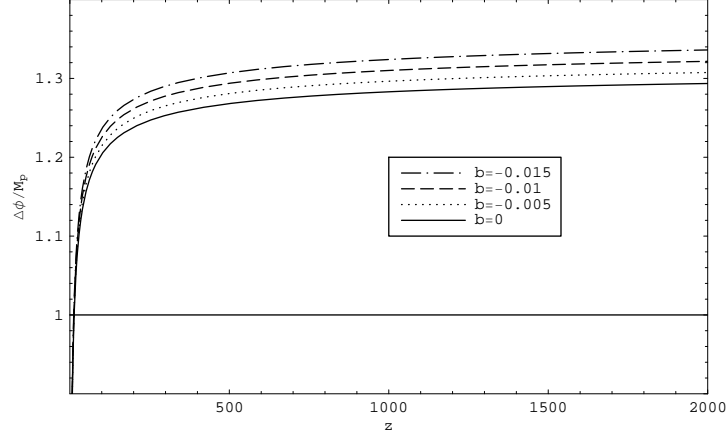


Fig. 3:  $\Delta\phi(z)/M_p$  versus the redshift  $z$  in the holographic quintessence model with the interaction term  $Q = Q_2$  in the flat universe for fixed  $\Omega_{m0} = 0.34$  and different  $b$ .

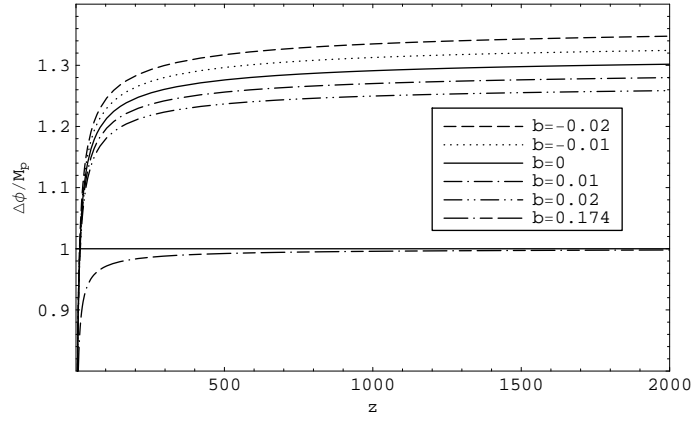


Fig. 4:  $\Delta\phi(z)/M_p$  versus the redshift  $z$  in the holographic quintessence model with the interaction term  $Q = Q_3$  in the flat universe for fixed  $\Omega_{m0} = 0.34$  and different  $b$ .

Then  $b > 0$  is also in the realistic physical region since  $\rho_m$  will never become negative in the case. But we find even for positive  $b$ , in the range of  $b$  given in FIG.2 in Ref.[31], Eq.(36) can not be satisfied yet. Our results are shown in Fig.4. Naively, in order to match the weak gravity conjecture, we should take  $b \gtrsim 0.174$  which is far beyond the range of  $b$  given in FIG.2 in Ref.[31].

In the subsection, we still fix  $\Omega_{m0} = 0.34$  which is slightly bigger than the maximum value of  $\Omega_{m0}$  displayed in FIG.2 in Ref.[31], since smaller  $\Omega_{m0}$  would make it more difficult for Eq.(36) to be satisfied.

### 3.3 the holographic quintessence model with interaction and spatial curvature

When both the interaction and spatial curvature are considered, we still have Eqs.(36) and (39). Of course, now the Friedmann equation includes the spatial curvature:

$$1 + \Omega_k = \Omega_m + \Omega_D.$$

The analysis of the effects of  $\Omega_{m0}$ ,  $\Omega_{k0}$  and  $b$  still works in the models with interaction and spatial curvature: larger  $\Omega_{m0}$ , smaller  $\Omega_{k0}$  or larger  $b$  will make it easier for the weak gravity conjecture to be satisfied. From FIG.5 in [31], we know  $b$  and  $\Omega_{k0}$  is anti-correlated: larger  $b$  corresponds to smaller  $\Omega_{k0}$ . Then there might exit the combinations of  $\Omega_{m0}$ ,  $\Omega_{k0}$  and  $b$  that satisfies the weak gravity conjecture Eq.(36).

However, unfortunately, for the models with the interaction term  $Q = Q_1$  or  $Q = Q_2$  in the non-flat universe, we can not find any combination of  $\Omega_{m0}$ ,  $\Omega_{k0}$  and  $b$  within the parameter space given in FIG.5 in Ref.[31] so that Eq.(36) is satisfied. We display our results in Fig.5 and Fig.6. In the two figures, we fix  $\Omega_{m0} = 0.35$  and  $b = 0$ , since smaller  $\Omega_{m0}$  or negative  $b$  would make it more difficult for Eq.(36) to be satisfied, and larger  $\Omega_{m0}$  or positive  $b$  are not physical.

For the models with the interaction term  $Q = Q_3$ , We find that the weak gravity conjecture can not be satisfied within the parameter space displayed in FIG.5 in [31] if  $b = 0$ . However, if  $b \gtrsim 0.04$  and  $\Omega_{k0} \lesssim -0.10$ , the combinations of  $b$  and  $\Omega_{k0}$  within the parameter space in FIG.5 of Ref.[31] which satisfy Eq.(36) can be found. Our results are shown in Fig.7.

## 4 Conclusion

In the paper, we have discussed the theoretical limits on the holographic quintessence model of dark energy from the weak gravity conjecture. Since the non-interacting holographic quintessence model without spatial curvature has been investigated in [29, 30], here we consider the other three cases separately: without interaction in the non-flat universe; with interaction in the flat universe; with interaction in the non-flat universe. Here, we use the observational constraints given in [31].

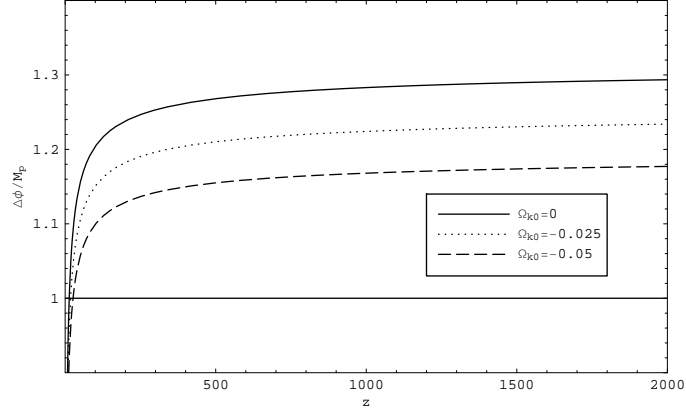


Fig. 5:  $\Delta\phi(z)/M_p$  versus the redshift  $z$  in the holographic quintessence model with the interaction term  $Q = Q_1$  and spatial curvature for  $\Omega_{m0} = 0.35$ ,  $b = 0$  and different  $\Omega_{k0}$ .

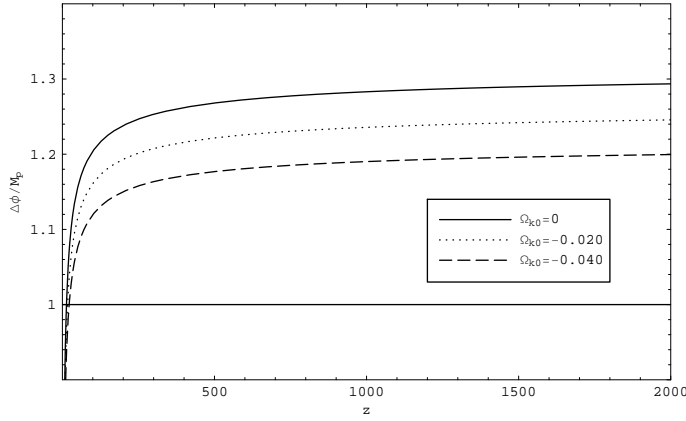


Fig. 6:  $\Delta\phi(z)/M_p$  versus the redshift  $z$  in the holographic quintessence model with the interaction term  $Q = Q_2$  and spatial curvature for  $\Omega_{m0} = 0.35$ ,  $b = 0$  and different  $\Omega_{k0}$ .

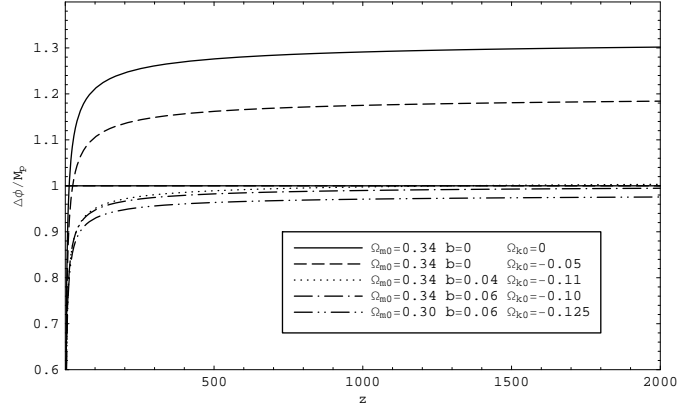


Fig. 7:  $\Delta\phi(z)/M_p$  versus the redshift  $z$  in the holographic quintessence model with the interaction term  $Q = Q_1$  and spatial curvature for different  $\Omega_{m0}$ ,  $b$  and  $\Omega_{k0}$ .

We find that the weak gravity conjecture can not be satisfied even in the the holographic quintessence models with interaction in the flat universe or the models without interaction in the non-flat universe. In [31], it is shown that the parameter space is amplified when simultaneously considering the interaction and spatial curvature. Then we might expect that it should be possible for the weak gravity conjecture to be satisfied in the models with interaction and spatial curvature.

However, we find that the models with the interaction term  $Q = Q_1$  or  $Q = Q_2$  in the non-flat universe are still inconsistent with the weak gravity conjecture within the parameter space in [31]. Fortunately, we find that, in the models with the spatial curvature and interaction term  $Q = Q_3$ , it is possible for the weak gravity conjecture to be satisfied. A roughly necessary condition for the weak gravity conjecture to be satisfied is shown:  $b \gtrsim 0.04$  and  $\Omega_{k0} \lesssim -0.10$ .

Then our results indicate that only the holographic dark energy models with the spatial curvature and interaction term  $Q = Q_3$  may be described by a consistent low-energy effective scalar field theory. And it seems that the weak gravity conjecture favors an open universe and the decaying of matter into dark energy.

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